

**Math 223 - Multivariable Calculus**  
Practice Exam 1

Name:

Please be sure to neatly **show and explain all of your work** and clearly label your answers. This is a closed-book, closed-notebook exam. Calculators are not allowed.

Please write and sign the Honor Pledge here when you are done:

Signed:

| Problem | Points | .   |
|---------|--------|-----|
| 1       |        | /12 |
| 2       |        | /12 |
| 3       |        | /12 |
| 4       |        | /10 |
| 5       |        | /12 |
| 6       |        | /10 |
| 7       |        | /12 |
| Total   |        | /80 |

1. Determine the equation of the plane containing the line

$$\vec{r}(t) = \langle 3 + 5t, 2t, 7 - 3t \rangle$$

and the point  $(3, 1, 9)$ . Show all work.

2. (a) Please compute the following limit or explain clearly why it does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{y}{x}$$

- (b) If  $f(x, y, z) = e^{yz} \sin(xy)$ , please compute  $f_{yz}$ .

3. The graph of the function  $f(x, y) = 4x^2 + y^2$  intersects the plane  $y = 2$  in a curve.
- (a) Draw the graph of the function and the curve of intersection.
- (b) Find the vector equation for the tangent line to the curve at the point  $(1, 2, 8)$ .

4. Please state whether each statement is true or false. If a statement is true, give an explanation why it is true. If it is false, give a counterexample or reason that shows that it is false.

(a) Suppose that  $\bar{v}$ ,  $\bar{x}$ , and  $\bar{w}$  are vectors in  $\mathbb{R}^3$ . If  $\text{proj}_{\bar{w}}\bar{v} = \text{proj}_{\bar{w}}\bar{x}$ , then  $\bar{v} = \bar{x}$ .

(b) Suppose that  $\bar{u}$  and  $\bar{v}$  are vectors in  $\mathbb{R}^3$ . If  $\bar{u} \times \bar{v} = \bar{0}$ , then either  $\bar{u} = \bar{0}$  or  $\bar{v} = \bar{0}$ .

5. (a) Let  $\bar{u} = \langle 6, -2 \rangle$  and  $\bar{v} = \langle 2, 1 \rangle$ . Compute  $\text{proj}_{\bar{v}} \bar{u}$ , the projection of  $\bar{u}$  onto  $\bar{v}$ .

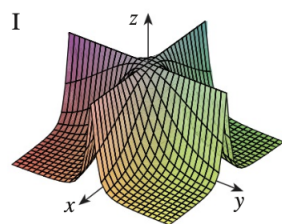
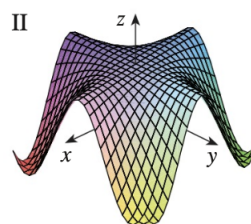
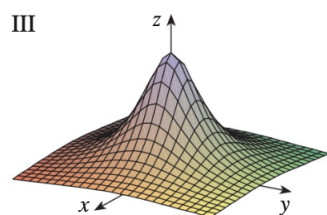
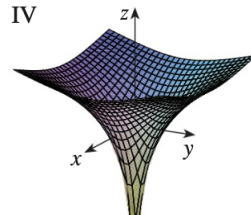
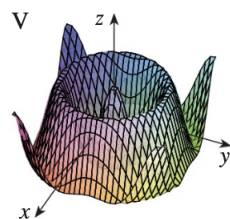
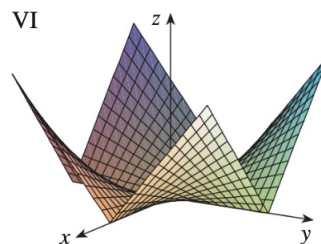
- (b) Use your answer to Part (a) to find the distance from the point  $(6, -2)$  to the line through the origin with direction vector  $\langle 2, 1 \rangle$ . Show all work and explain your reasoning.

(Note: you do not need to know a formula here. It is possible to work out this answer directly using the facts about vectors that you know.)

6. Please indicate which of the following represents a graph of

$$f(x, y) = \frac{1}{1 + x^2 y^2}.$$

Explain your reasoning.



7. At what point on the graph of

$$f(x, y) = x^2 + 3y^2$$

is the tangent plane parallel to the plane  $4x - 6y + z = 7$ ?

Please show all work and explain your reasoning.