

Tangent Plane / Linearization Computation

Ex $f(x,y) = x^2 + y^2$

Tangent plane at $(1,2)$? Linearization at $(1,2)$?

$$f_x(x,y) = 2x$$

$$f_x(1,2) = 2$$

$$f_y(x,y) = 2y$$

$$f_y(1,2) = 4$$

normal vector: $\begin{matrix} f_x(1,2) & f_y(1,2) & \swarrow \text{always } -1 \\ \downarrow & \downarrow & \\ \langle 2, 4, -1 \rangle \end{matrix}$

point: $(1, 2, 1^2 + 2^2) = (1, 2, 5)$

thus the tangent plane is given by:

$$\langle 2, 4, -1 \rangle \cdot \langle x-1, y-2, z-5 \rangle = 0$$

typical format
↓ for plane

$$2(x-1) + 4(y-2) - (z-5) = 0$$

$$2x + 4y - z = 5$$

... or ...

equivalent formulation that highlights linearization

$$z = 5 + \underbrace{2(x-1) + 4(y-2)}_{L(x,y)}$$